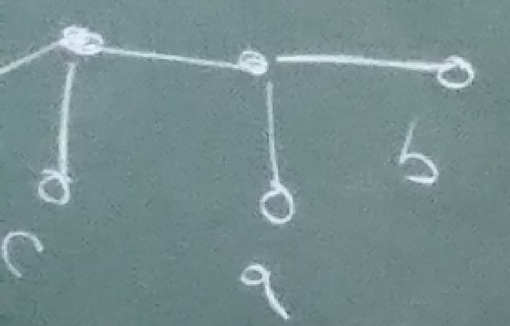
Π_{F_b}

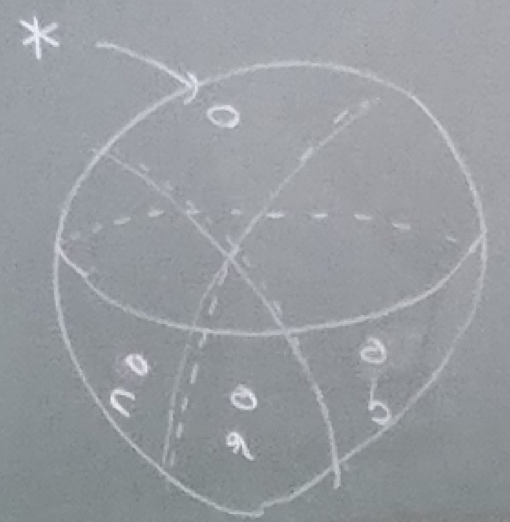
$\mathcal{Z}^{log}$ over $\mathbb{C}$



Since $\Pi_{F_b} \overset{p_2}{\xrightarrow{\sim}} \Pi_1$, $\Pi_{F_c} \overset{p_2}{\xrightarrow{\sim}} \Pi_1$ and $\alpha_2 = id$

we have $\alpha|_{\Pi_{F_b}} = id$, $\alpha|_{\Pi_{F_c}} = id$

Again by considering the geom gen fiber of $X_2 \to X_1$,



$$\Pi_{2/1} \cong \varinjlim \left(\Pi_{F_b} \hookleftarrow I_a \hookrightarrow \Pi_{F_c} \right)$$

[cf. van Kampen]

$$\alpha|=id \qquad \circlearrowleft \alpha \qquad \circlearrowleft \alpha_1 = id$$
$$1 \to \Pi_{2/1} \to \Pi_2 \to \Pi_1 \to 1$$

$\therefore$ $\alpha|_{\Pi_{2/1}} = id$

Since $\alpha_1 = id$, it follows that $\alpha = id$ //

$\Pi_{2/1}$: slim

Next month
2/16
13:00–
K. Nakamura

Hence, by replacing

We may as

Next, by considering

$\Pi_{2/1} \cong \Pi$ (

$\Pi_{F_b} \subseteq \Pi_{2/1}$:

(a

[§4]

Thm 2 (affine case)

Suppose that X/k: affine hyperbolic curve of type $(g,r) \neq (0,3)$

Then (1) $\Pi_2 \hookrightarrow \mathrm{Aut}^{IFC}(\Pi_2)$ bij

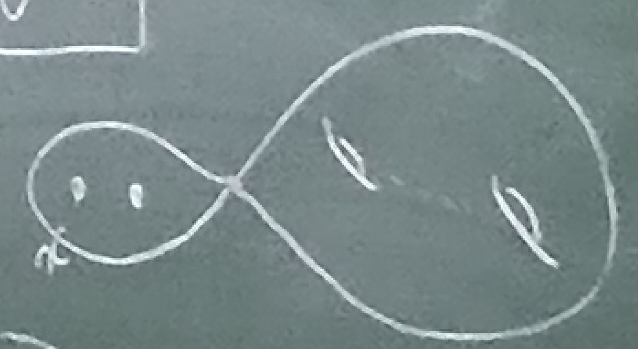
(2) $\mathrm{Out}^{FC}(\Pi_2) \longrightarrow \mathrm{Out}^{FC}(\Pi_1)$ inj

(∵) (1) $\Rightarrow$ (2): the same arguments as "Thm 1 (1) $\Rightarrow$ (2)"

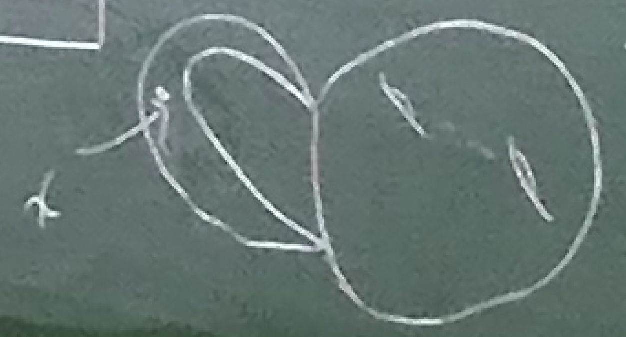
So we consider (1)

For simplicity, we assume $r \geq 2$ [$W^{\log}$

degenerate $z^{\log}$ degenerate


]

[$r=1$: $W^{\log}$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Next month
2/16
13:00–
K.Nakamura
Room 306

$z_n^{\log}$: n-th log

$1 \to$

$\alpha \in \mathrm{Aut}^{IFC}(\Pi_2)$

Now, by considering the fibe

$\Pi_{2/1} \cong \Pi$

"Π_{E_x}" $\subseteq$ "Π_{V_x}"

"Π_{F_x}" $\subseteq$

E_x

...perbolic curve

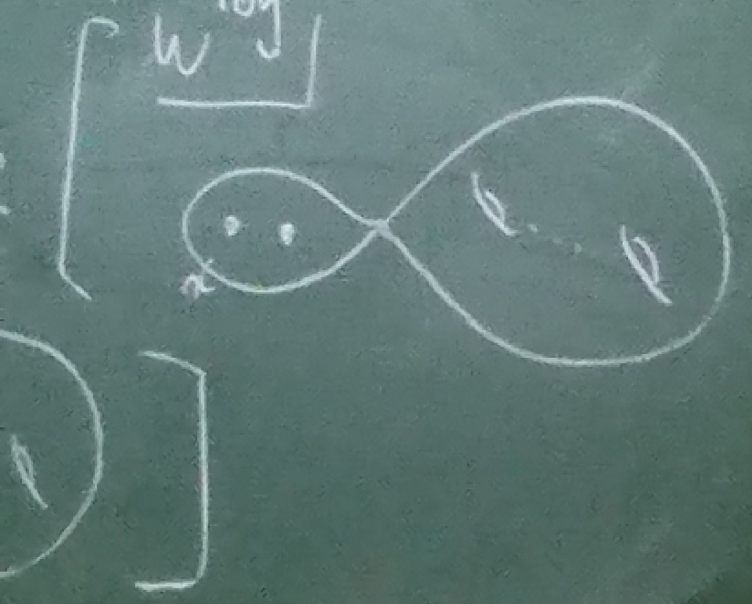
$(g,r) \neq (0,3)$

inj

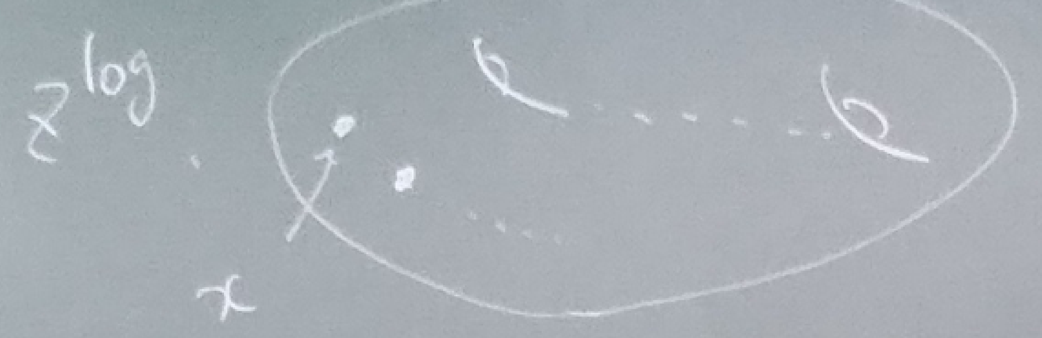
...nts

"... 1 (1) $\Rightarrow$ (2)"

degenerating



Let $Z^{\log}$: smooth log curve associated to X



Next month
2/16
13:00-
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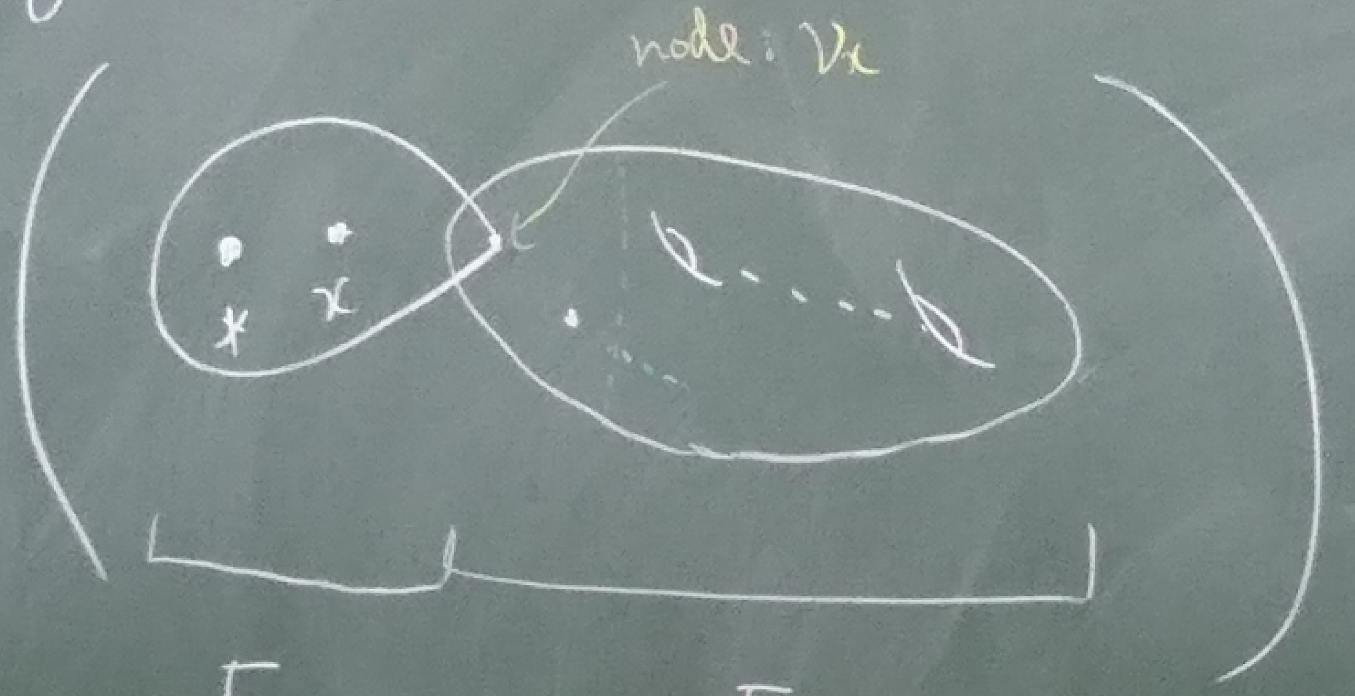
Room 006

$Z_n^{\log}$: n-th log config sp of $Z^{\log}$

$\alpha \in \mathrm{Aut}^{\mathrm{IFC}}(\Pi_2)$ $\qquad 1 \to \Pi_{2/1} \to \overset{\circlearrowleft \alpha}{\Pi_2} \to \overset{\circlearrowleft \alpha_1}{\Pi_1} \to 1$

Now, by considering the fiber of $Z_2^{\log} \to Z^{\log}$ over x

$\Pi_{2/1} \cong \Pi\Big($  $\Big)$

E_x $\qquad$ F_x

"Π_{E_x}" $\subseteq$

"Π_{F_x}" $\subseteq$

$\ni$ "Π_{v_x}"

Hence, by replac...

We w...

Next, by consid...

By C...

By Comb GC, since

vertical subgp $\longmapsto$ vertical subgp

edge-like " $\longmapsto$ edge-like "

it follows that

" α induces identity permutation

on the set of conj classes of $\Pi_{E_x}, \Pi_{F_x}, \Pi_{v_x}$

Now we fix $\Pi_{v_x} \subseteq \Pi_{Z_1}$

[among its Π_{Z_1}-conj]

Π_{E_x}, Π_{F_x}: two vertical subgps $\subseteq \Pi_{Z_1}$ containing Π_{v_x}

$\therefore \quad \alpha(\Pi_{v_x}) = \xi^{-1} \Pi_{v_x} \xi \quad (\exists \xi \in \Pi_{Z_1})$

$$\Rightarrow \quad p_2 \alpha(\Pi_{v_x}) = p_2(\xi)^{-1} p_2(\Pi_{v_x}) p_2(\xi)$$

$$p_2 \alpha(\Pi_{v_x}) = p_2(\Pi_{v_x})$$

$$\Rightarrow \quad p_2(\xi) \in p_2(\Pi_{v_x})$$

Prop1. (2)

[illegible] terminal

Thus, by replacing "ξ" by an appropriate element,

we may assume that $\xi \in \boxed{\Pi}_{12}$

(Moreover, by replacing "α" by "$\mathrm{Inn}(\xi^{-1}) \circ \alpha$",

we may assume that $\boxed{\alpha(\Pi_{v_x}) = \Pi_{v_x}}$

$$\Pi_{v_x} \subseteq \alpha(\Pi_{E_x}) = \Pi_{E_x} \quad (1)$$

$$\Pi_{v_x} \subseteq \alpha(\Pi_{F_x}) = \Pi_{F_x}$$

Since $\begin{pmatrix} p_2 : \Pi_{F_x} \twoheadrightarrow \Pi_1 \\ \text{and } \alpha_2 = id \end{pmatrix}$, we have $\alpha|_{\Pi_{F_x}} = id$ (2)

Next month
2/16
13:00 ~
K. Nakamura
Room 006

We may replac

" $W^{\log}_x :=$

[where $\Pi_n \cong$ k

Thus, by considering

$$\Pi_{2/1} \cong \Pi \Big($$

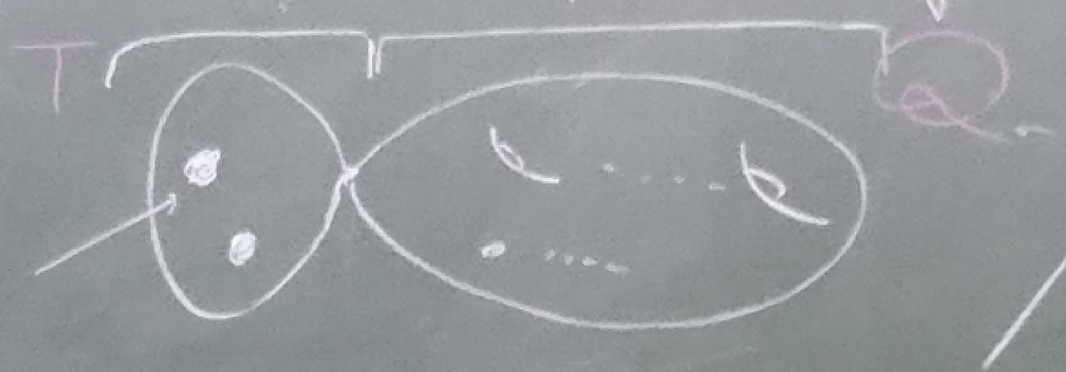
...propriate element,

Next month
2/16
13:00-
K. Nakamura
Room 306

Here, we note that, by applying

(• deformation theory of stable log curves
 • specialization thm of log fund gps),

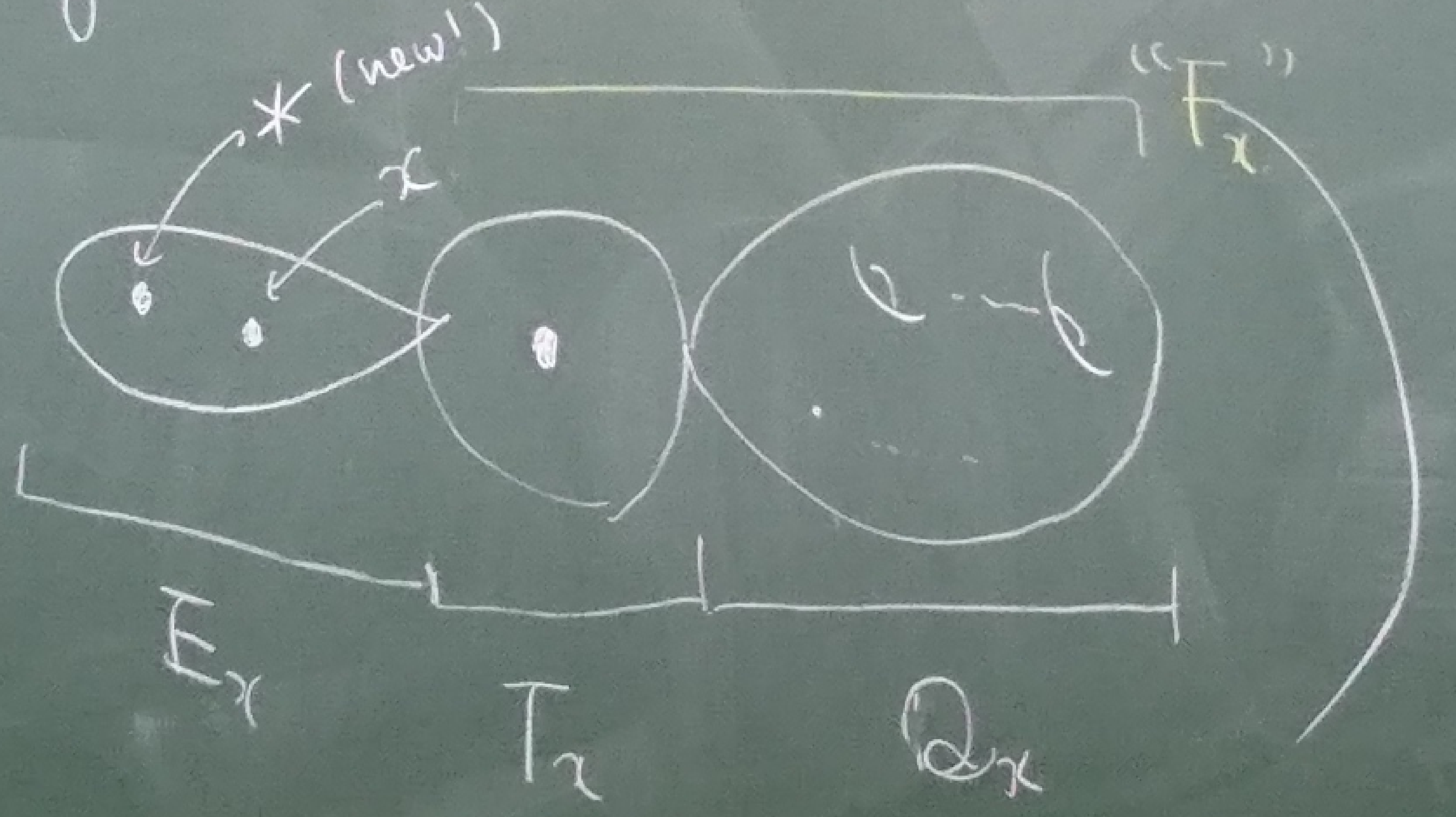
we may replace "$Z^{\log}/k$" by

"$W^{\log} :=$ 

$/ \ S^{\log} = (\mathrm{Spec}\, L, \mathbb{N})$" $L = \bar{L}$, $\mathrm{ch}(L) = 0$

[where $\Pi_n \cong \ker(\pi_1(W^{\log}) \to \pi_1(S^{\log}))$]

Thus, by considering the fiber of $W_2^{\log} \to W^{\log}$ over x

$\Pi_{2/1} \cong \Pi\Big($ 

$\Big)$

Π_{v_2}

Π_{E_x} — ①

Π_{F_x}

②

$|_{\Pi_{F_x}} = id$

Now we fix $\Pi_{v_x} \subseteq$ ...

(among its $\Pi_{2/1}$- ...

∴ $\alpha(\Pi_{v_x}) = \xi^{-1}$ ...

$\Rightarrow p_2\,\alpha(\Pi_{v_2}) = p_2($ ...

$\parallel$

$p_2(\Pi_{v_x})$

⇝ By ②,

g curves

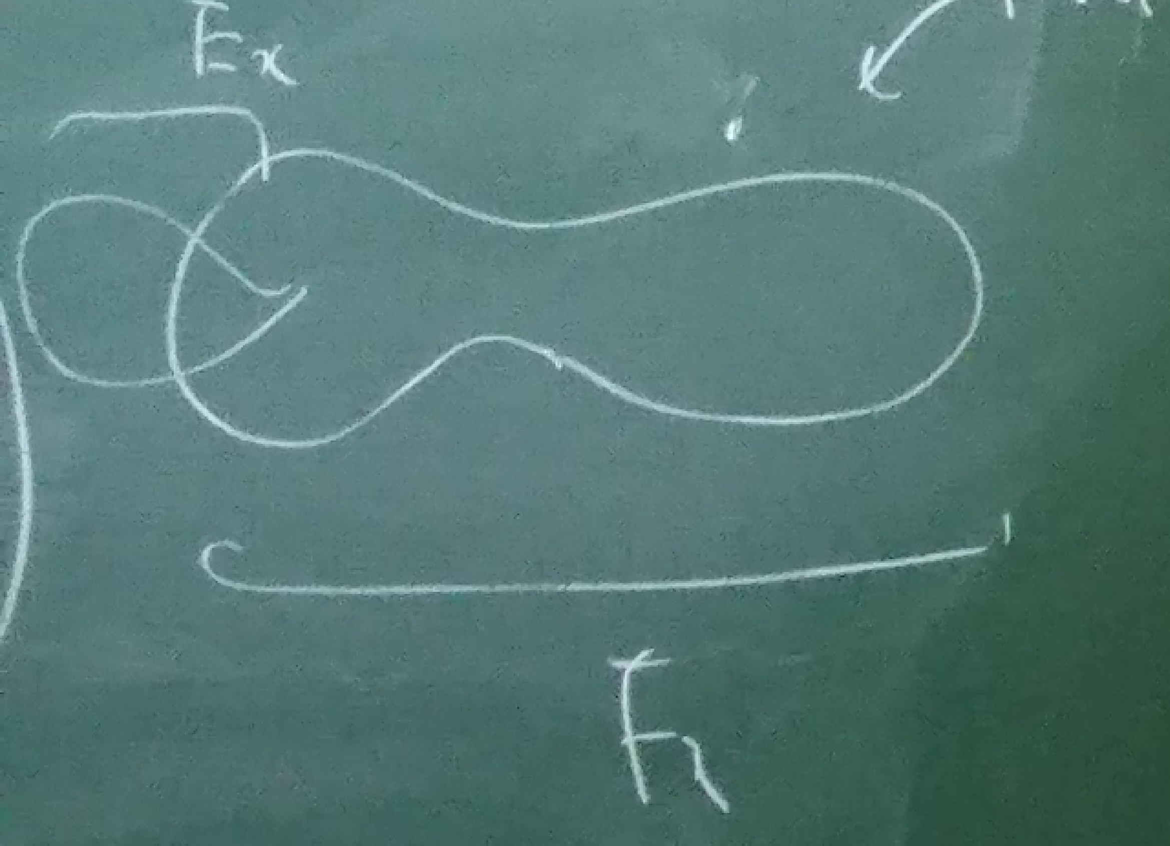
nd g, r

)

$L = \bar{L}$ $\quad ch(L) = 0$

og $= (\mathrm{Spec}\, L, \mathbb{N})$

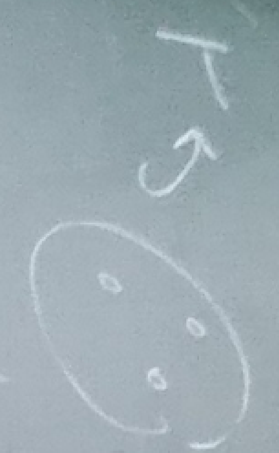
g over X



$\rightsquigarrow$ By ②, $\alpha|_{\Pi_{T_x}} = \mathrm{id}$ — ③

Moreover, let

$T^{\log}/S^{\log}$: smooth log curve determined by $U_T =$



$T_n^{\log}$: n-th log config sp

Π_n^{tpd} : $\ker(\pi_1(T_n^{\log}) \twoheadrightarrow \pi_1(S^{\log}))$

$\rightsquigarrow$ $1 \longrightarrow \overset{\alpha \curvearrowright}{\Pi_{2/1}^{tpd}} \longrightarrow \Pi_2^{tpd} \longrightarrow \Pi_1^{tpd} \longrightarrow 1$

Thus, by ①, ③ $\quad \alpha(\Pi_{2/1}^{tpd}) = \Pi_{2/1}^{tpd}$

In fact, α arises from $\exists\, \alpha^{tpd} \in \mathrm{Aut}^{IFC}(\Pi_2^{tpd})$

$\uparrow$ Thm 1.(i)

$\boxed{H}_2^{tpd}$

$\rightsquigarrow$ α^{tpd} : $\boxed{H}_{12}^{tpd}$-inner

$\rightsquigarrow$ $\alpha|_{\Pi_{2/1}^{tpd}}$: $\exists\ \Pi_{2/1}^{tpd}$-inner — ④

lem 2 $G \supseteq H$ closed subgp s.t. $N_G(H) = H$ and $Z(H) = \{1\}$

Let $\beta \in \mathrm{Inn}(G)$ s.t. $\beta|_H = \mathrm{id}_H$

$\Longrightarrow \beta = \mathrm{id}$

Next month
2/16
13:00 ~
K. Nakamura
Room 306

We may

" log
W

[where Π

Thus, by consid

$\Pi_{Z_1} \cong \Pi$

In light of lem 2, prop 1, 3, 4,

we conclude that $\alpha|_{\Pi_{Z_1}^{\text{tpd}}} = \mathrm{id}$

$\Pi_{Z_1} \cong \varinjlim(\Pi_{E_1} \leftarrow \Pi_{\mathbb{A}} \rightarrow \Pi_{E_2})$

$\therefore \alpha|_{\Pi_{E_1}} = \mathrm{id}$

Since $\alpha|_{\Pi_{E_2}} = \mathrm{id}$, by applying "van Kampen", it follows that $\alpha|_{\Pi_{Z_1}} = \mathrm{id}$

$\therefore \alpha = \mathrm{id}$ //